Chapter 5: Theory of Production

The production function indicates the physical technological relationship between input and output. It shows that with a given state of technological knowledge during a particular period of time, the maximum possible output that can be produced with a given amount of input or alternatives. Minimum quantity of inputs necessary to produce a given level of output.

Some basic points to be noted with respect to the production function are as follows:

1. The production function is expressed with reference to the particular period of time.
2. It establishes a functional relationship between physical input and output.
3. It assumes a given state of technology. Improvement in technology creates a new production function that produces a larger output with the same amount of inputs.
4. It shows a flow of output resulting from a flow of input.

Production function can be explained as follows:

\[ Q = f(L, K, N, T, ...) \]

And, function thus shows production is the result of the various inputs such as land, labour, capital, etc., which go into the production of commodity X.

For simplicity, we can take only 2 most essential factors in production function: labour (L) and capital (K).

\[ Q = f(L, K) \]
Thus, two inputs are substitute to some extend. There are 2 types of production function.

1. Short-run production function / Single variable production function.
   It is the study of the change in one variable factor on the output where other factors are fixed. In the short-run, output can be increased only by increasing the variable factor like labour, raw materials, etc. Other fixed factors such as land, building, machine, etc. can’t be varied. Law of variable proportions studies the short-run input-output relation.

2. Long-run production function / Multi-variable production function.
   It is the study of the effect of change in all inputs on outputs in the long-run. Thus, in the long run all inputs are variable. It is expressed by law of return to scale. It refers to the functional relationship between the quantities of all inputs & output.

Cobb-Douglas Production Function

In 1928 American Scholars C.W. Cobb & P.H. Douglas made a statistical enquiry into some manufacturing industries in America & other countries & studied the empirical relation between change in physical input & resulting in physical output. From their studies, they introduce a production function with two variable inputs, labour & capital. This production function is known as Cobb-Douglas Production Function.
\[ Q = AL^a K^b \]  or \[ Q = AL^\alpha K^{1-a} \]
\[ Q = AL^a K^b \]  or \[ Q = AL^\alpha K^{(1-a)} \]

where, \( Q \) = Total output of the product
\( A \) = Input productivity efficiency parameter
\( L \) = Quantity of labour employed
\( K \) = Quantity of capital employed
\( a \) = 'b' or \( a' \) and \( b' \) = Exponents / Powers.

The constant parameters of the production function which measures the responsiveness of output to change in labour & capital or output elasticity of labour & capital with given technology.

The sum of the power of inputs \((a+b)\) measures returns to scale. If \((a+b)=1\) then returns to scale are constant. Also known as Linear Homogeneous P.F.

If \((a+b) > 1\) then returns to scale are increasing.

If \((a+b) < 1\) then returns to scale are decreasing.

Considered the production function \((P.F.)\)

\[ Q = 5 L^{0.5} K^{0.5} \]

Does it represent increasing, decreasing or constant returns to scale. So it

Inc. both labour & capital input by \( m \).

we have:

\[ Q = 5 (mL)^{0.5} (mk)^{0.5} \]
\[ = 5 \times m^{0.5} x L^{0.5} x m^{0.3} x k^{0.3} \]
\[ = 5 \times m^{0.8} x L^{0.5} x k^{0.3} \]
\[ = m^{0.8} (L^{0.5} x k^{0.3})^{5} \]
\[ = m^{0.8} Q \leftarrow \text{from 0} \]

Thus, by increasing both \( L \) & \( K \) by \( m^{0.8} \) it is the case of decreasing returns to scale.
Total Production (TP), Marginal Production (MP) & Average Production (AP)

The quantity of good produced by using factors or inputs can be expressed on the basis of TP, MP & AP.

\[ TP \rightarrow \text{Total volume of goods produced during the specified period of time is known as TP.} \]
\[ TP = AP \times \text{Units of variable factor} \]
\[ \text{or, } TP = EMP \]

MP \rightarrow \text{It is the addition to the TP due to a unit increase in variable factor.}
\[ MP = TP(n+1) - TP_n \]
\[ \text{or, } MP = \Delta TP \]
\[ \text{A Variable factor} \]
\[ AP = \frac{TP}{\text{Units of variable factor}} \]

Law of Variable Proportions

The law of variable proportions concerns with short-run production function. This law examines the production with one factor variable, keeping the quantities of other factor fixed. TP increases at an increasing rate, but after a point, increases at a decreasing rate becomes maximum & starts to decline.
<table>
<thead>
<tr>
<th>Land</th>
<th>Unit of Labour</th>
<th>TP</th>
<th>MP</th>
<th>AP</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>10</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>18</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>24</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>28</td>
<td>4</td>
<td>5.6</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>30</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>30</td>
<td>0</td>
<td>4.3</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>28</td>
<td>-2</td>
<td>3.5</td>
</tr>
</tbody>
</table>

In the table, land is a fixed factor & labour is a variable factor. With a given fixed qty of land as a farmer raises employment of labour from one unit to 6th unit, TP increases from 1 unit to 6th unit & become maximum at 7th unit then TP starts to decline. This fact can be viewed clearly from MP column. MP upto 3rd unit of labour increases & starts to diminish & at 7th unit it become zero & then it is negative. Hence, TP increases at an increasing rate up to 3rd unit and the increase at a decreasing rate upto 6th unit. Becomes maximum at unit 7 & then diminishes. The AP reach maximum at the 4th unit of labour & then starts to diminish.
Stage I (Increasing Return)

In this stage, TP increases at an increasing rate to a certain point and then rate of increasing switching from increasing to diminishing. TP is increasing up to point A at an increasing rate & then started to increase at diminishing rate. The AP increases throughout this stage & reached to maximum at point E. The boundary of this stage is point E where marginal & average products are equal.
Stage II

In this stage, TP continues to increase at a diminishing rate until it reaches its maximum at point c where second stage ends. In this stage, both the NP & AP are diminishing but are lower. At the end of second stage, i.e., at point F, NP is zero.

Stage III

This stage starts from the point C. In this stage, total product is declining & NP is negative & NP goes below the x-axis. In this stage, variable factor is too much relative to the fixed factor.
Chapter 5  Theory of Production

Causes of Increasing Returns in the First Stage

1. Increase in efficiency of fixed factor: In the initial stage of production, the quantity of fixed factor are large than that of variable factor. Therefore, when more and more units of the variable factor are used to the constant quantity of the fixed factor, the fixed factor is more efficiently utilized so, the efficiency of the fixed factor increase as additional units of the variable factor are added to it. This causes the production to increase at a rapid rate. At the beginning, when the variable factor is relatively smaller in quantity, some amount of fixed factor may remain unutilized. Therefore, when the variable factor is increased, the fuller utilization of the fixed factor becomes possible. As a result, the increasing returns are obtained.

2. Increase in the efficiency or productivity of variable factor: As variable factors are increased in the first stage, the efficiency or productivity capacity of variable factor itself increases. And in result, it becomes possible to have division of labour which increased the efficiency of variable factors. Thus, in the initial stage production increases.

Causes of Diminishing Returns in the Second Stage

1. Indivisibility of the fixed factor: As more and more quantities of variable factors are added, the combination of fixed factor and variable factors will be in the best proportion at a certain level of output. This is the optimum factor of proportion. But
beyond this point a quantities of variable factor are further added, the efficiency of the fixed indivisible factor decreases. AP diminishes & MP also goes on diminishing & become zero.

2. Imperfect Substitutability of the factor: According to John Robinson, factors can be substituted upto a limit beyond this limit if more variable factor is added to the fixed factor diminishing returns sets in imperfection substitutability.

Causes of Negative Returns in the Third Stage.

1. Excessive variable factor: This stage sets in because quantity of variable factor becomes too excessive relative to the fixed factor. Too large no. of variable factor reduces the efficiency. Hence, MP of variable factors becomes negative & TP decline.

Stage of Operation

A rational producer will never choose to produce in Stage III where MP of variable factor is -ve. Even if the variable factor is free, a rational producer will not produce in Stage III coz TP declines in this stage.

A rational producer also doesn’t choose to produce in Stage I although TP increases & MP of variable factor is +ve in this stage. This is coz the MP of fixed factor in this stage is -ve. Thus, there is opportunities of increasing prod. by increasing quantity of the variable factor where AP is continue to rise throughout the stage I.
Thus, it is clear that a rational producer will produce in stage II where both MP & AP of variable factors are diminishing. At which particular point in this stage, the producer will decide to produce depends upon the prices of factors.

Relation between MP & AP.

AP & MP both are increasing. MP is highest at point A. AP is maximum at point M where MP & AP are equal & coincide each other. MP is less when AP is diminished. MP becomes -ve but AP remains +ve.

Suppose a production schedule is given as follows. Complete the table & find the following.

<table>
<thead>
<tr>
<th>Labour</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP</td>
<td>0</td>
<td>4</td>
<td>10</td>
<td>18</td>
<td>24</td>
<td>28</td>
<td>30</td>
<td>30</td>
<td>28</td>
</tr>
</tbody>
</table>

(a) Output with the highest AP. (b) Employment at which MP = AP. (c) Employment at which TP is highest. (d) Construct a fig. (e) Enumerate law of variable proportion based on the fig. (f) Exp. the relationship between MP & AP.
<table>
<thead>
<tr>
<th>Labour</th>
<th>TP</th>
<th>AP</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>5.5</td>
<td>8.4</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>4.2</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
<td>3.5</td>
<td>-2</td>
</tr>
</tbody>
</table>

a) From the table, output with the highest AP is 6 units of Labour.
b) Employment at which MP = AP is 4 units of Labour.
c) " " " " TP is highest & constant 18.7 units of Labour.
d)
c. In the fig. units of labour is measured along X-axis & TP, AP & MP are measured along Y-axis. TP starts to increase at increasing rate from the begg beginning upto 3rd unit of labour. Then TP increase at decreasing rate upto 6th where MP is 0 unit of labour. Become maximum at 7th unit then TP starts to fall. The AP reaches maximum at 4th unit of labour & the AP starts to diminish but never attain 0 & -ve but MP attain zero & also reaches -ve.

b. Relationship betw. AP & MP
i. When MP > AP, AP is rising.
ii. When MP = AP, AP is at its maximum.
iii. When MP < AP, AP is falling.

Iso-quant, Iso-product curve, Equal-product curve
The term 'isocost' has been derived from a Greek word 'iso' means equal & Latin word 'quant' means quantity. Therefore, the isocost curve is also known as equal production curve & prod. indifference curve. An isoquant is the locus of points representing different combinations of two inputs (labour & capital) yielding the same level of output. The producer is indifferent among them.

<table>
<thead>
<tr>
<th>Combination</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour (L)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Capital (K)</td>
<td>9</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

The schedule indicates, for the production of 10 units of X A or B or C or D combi. of L & K can be used, if we plot all these combi. graphically we get a curve known as iso-curve.
In the fig. points A, B, C, & D on the isoquant I_Q shows four different combinations of capital & labour. But all these combinations yield the same output i.e. 10 units. Movement from A to B indicated decreasing quantity of K & increasing quantity of L. This implies substitution of labour for capital yield the same level of output along the isoquant I_Q.

Iso-quant Map

The fig. shows a set of isoquant. The combinations lying on the same isoquant shows an equal level of output. But the higher isoquant represents the higher level of output than the lower isoquant.
Properties of Isoquants

1. Slope of Isoquant curve is negative or downward sloping

   An isoquant curve is negatively sloped. This is so because when unit of labor is increased, the units of capital must be reduced so as to keep the amount of output constant.

![Diagram showing isoquant curve]

In the fig, the movement from A to B on IQ shows that if unit of capital is reduced from the production process, unit of labor is increased to maintain to some level of output.

2. Isoquant curve is convex to the origin

   All the isoquant curves are convex to the origin. It is due to the fact that MRTS falls as more and more unit of labor is substitute for capital. It means, less and less unit of K is substituted by labor so as to keep the level of output unchanged.

![Diagram showing convex isoquant curve]
In the fig., units of labour are increased & unit of capital are decreased. But both labour & capital are increasing & decreasing in same rate respectively. It means that initially more & less & less unit of K are substituted for each unit of L. This gives us a convex IQ curve.

3. IQ curve never intersect or tangent to each other

Two IQ curves never intersect each other. To prove this property, let’s suppose that these two curves intersect each other.

In the fig., two IQ curves IQ1 & IQ2 are intersected each other at point A. Point A & B lie on the same IQ curve IQ2. They represent same level of output.

Combi. A = Combi. B \(\ell\)

Similarly, points A & C lie on the IQ1 curve showing equal level of output.

Combi. A = Combi. C \(\ell\)

From i & ii

Combi. C = Combi. B (coz B & C are equal to A)
But this result is absurd since B lies on the higher IQ curve. i.e., IQ_2 thus represents higher level of output. While point C is located on the lower IQ, i.e., IQ_2 shows the smaller amount of output. In this way, B can't be equal to C. Hence, too IQ curves never intersect or tangent to each other.

4. Higher Iso-quant represents higher level of output

The higher IQ curve represents higher level of output than the lower one because higher IQ consists of more of both labours & capital.

In the fig., the higher IQ curve IQ_2 always represents a higher level of output than lower IQ_1. At any point on IQ_2 consists of more of either capital or labour or both. Therefore IQ_2 represents a higher level of output. For eg: 'a' is the combi. of K & L whereas 'b' is the combi. of K & 2L. Which clearly show that there is more amount of K & capital & labour in combi. b.
Iso-cost Line.

An Iso cost line shows all possible combination of two factors that the producer can get by spending a given amount of money on two factors, \( L \) & \( K \) given their prices.

It is the total cost using the total amount of money for spending \( L \) & \( K \).

Suppose a producer wants to spend Rs 200 on factor \( L \) & \( K \). Price of \( L \) is Rs 20 & \( K \) is Rs 40.

\[
C = P_L \times Q_L + P_K \times Q_K
\]

\[
200 = 20 \times Q_L + 40 \times Q_K
\]

\[
\frac{200}{20} = Q_L
\]

\[
10 = Q_L
\]

\[
C = P_L \times Q_L + P_K \times Q_K
\]

\[
200 = 0 + 40 \times Q_K
\]

\[
S = Q_K
\]

If he spends all amount of money on getting factor \( L \), he can get 10 units of it.

If he spends all amount of money on getting factor \( K \), he can get 5 units of it.

But he get some other combination of \( L \) & \( K \) too. Such as \((2L + 4K), (4K + 3K), (6L + 2K)\).
Slope of Iso-cost line: The slope of Iso-quant line represents the price of the factor of Iso- k as factor L.

Slope of IC: \( \frac{P_L}{P_k} \)

Y-Intercept: \( OI = \frac{C}{P_k} - P_L \)

X-Intercept: \( OL = \frac{C}{P_L} - P_k \)

Shift in Iso-cost line:

The Iso-cost line shifts with the change in total outlay. If the price of labour (L) and capital (K) remain constant but the producer decides to spend Rs 300 rather than Rs 200, the Iso-cost line shifts outward to A1B1. It shows that now 60 units of capital or 30 units of labour can be purchased. Similarly, when the producer wants to spend Rs 400, the Iso-cost line shifts upward to A2B2. Therefore, if the prices of both inputs remain the same, but the expenditure to be done or the purchase of factors increases, we get a higher Iso-cost line. Similarly, if the proposed expenditure to be done by the producer falls, the Iso-cost line shifts downward but remains parallel to the original Iso-cost line.
The slope of the Iso-cost line can also change when the outlay remains the same but the price of one input or both the inputs changes. The change is similar to the rotation in the price line as was explained in the IC analysis.

Fill up the following IC Schedule

<table>
<thead>
<tr>
<th>Combination</th>
<th>Labour</th>
<th>Capital</th>
<th>MR TSLK</th>
<th>( \frac{\Delta K}{\Delta L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>4</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>10</td>
<td>4</td>
<td>( \frac{14-10}{2-1} )</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Least cost combination of Inputs
(Optimal Combination of Inputs)

1. Arithmetic method
2. Geometric method

1. Least cost of \( L \times K \) are calculated from total cost.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Labour</th>
<th>Capital</th>
<th>Total Cost of ( L )</th>
<th>Total Cost of ( K )</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>12</td>
<td>21</td>
<td>60</td>
<td>81</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>9</td>
<td>28</td>
<td>45</td>
<td>73</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>7</td>
<td>35</td>
<td>35</td>
<td>70</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
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<td>30</td>
<td>72</td>
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<td>7</td>
<td>5</td>
<td>49</td>
<td>25</td>
<td>74</td>
</tr>
</tbody>
</table>

Suppose a producer has decided to produce 100 units of \( \text{comm } X \). There are 5 different combinations of \( L \times K \). Per unit labour cost \( (P_L) = Rs 7 \) & capital cost \( (P_K) = Rs 5 \).

In the table, among the 5 alternative combinations, the producer will choose the combination \( C \), where 5 units of labour & 7 units of capital are capable of producing 100 units of \( \text{comm } X \) at the least cost of Rs 70. Thus, \( C \) is the least or optimal combination of \( L \times K \) for producing 100 units of \( \text{comm } X \).

2. A rational consumer always tries to maximize profit by producing a given output at least cost combination of factors. It is determined on the basis of iso-cost line & iso-quants. There are two essential conditions for determining least cost combination of factors.
a. Iso-cost line should be tangent to IQ. In other words slope of the Iso-cost line & IQ should be equal.

\[ \frac{\Delta K}{\Delta L} = \frac{P_K}{P_L} = \text{(Price of K)} / \text{(Price of L)} \]

b. NRTSUK must be diminishing i.e. IQ convex to origin at the point of equilibrium.

In the fig. K1L1, K2L2 & K3L3 are Iso cost line representing total cost < 200, 400 & 600 respectively. IQ are represented by IQ1 & IQ2 representing 5 unit & 10 units of commod X respectively. Among the diff. combination the producer chooses 'N' combination at which K2L2 is tangent to IQ2. Combi 'N' will cost him least for producing 10 units of commodity X. He will not choose any other combi. such as K & R coz it lies in higher Iso cost line K3L3, it means higher
The total cost Rs. 600 for producing 10 units of commodity X. The producer doesn't choose any other combination of L & K on IQ₁ because it represents less unit of commodity X (5 units). Therefore, the producer will be in equilibrium at the point N where the amount of money spent will be least by choosing 0X₁ of labour L & 0Y, amount of capital.

At the equilibrium point, the slope of IQ represented by NRTSₖ_L is equal to the slope of ISO cost line represented by Price ratio of two factors L & K.

At the equilibrium point, N_IQ is convex to the origin indicating decreasing NRTSₖ_L.

In the fig., NN & T are the equilibrium points representing 5 units, 10 units & 15 units respectively. By joining these equilibrium points we get a line known as scale line or expansion path. Scale line shows the points of tangency between ISO cost line & IQ. It indicates how the producer will change comb. of two factors L & K to expand the output of the firm. It shows the cheapest method of
Returns to factor \( \rightarrow \) law of variable proportion (change in proportion)

Returns to scale \( \rightarrow \) long run input-output relation (change in scale)

\[
\begin{align*}
2L + 1M & \quad \text{All the inputs are variable, No change in the proportion} \\
4L + 2M & \\
6L + 3M &
\end{align*}
\]

each of goods at the given prices of two factors.

Returns to scale:

Long run Input-Output relation

In the long run all the factors are variable distinction betweenfixed & variable factors doesn't exist. In long run by changing all the factors in the same (given) proportion the firm can change scale of production. When a firm change its scale of production what would be the real effect on the profit in the long run is explained by the returns to scale. Profit is subject to 3 types of Returns to Scale:

1. Increasing Returns to scale
2. Constant Returns to scale
3. Decreasing Returns to scale

1. Increasing Returns to Scale

Increasing returns to scale means increases in output a greater proportion than the increase in inputs. For e.g: if all inputs are increased by 25% & output increased by 30% then increasing returns to scale is said to exist. It can be shown by usingIsoquant in the figure.
The fig shows the prod. function with increasing returns to scale. The line OS drawn from the origin is the scale line which are the same slope throughout because L & K are increased in a given fixed proportion indicating the changes in the scale of production. IQ1, IQ2, IQ3 & IQ4 represents iso-quants indicating 50, 100, 150 & 200 units of output respectively. In the case of Increasing returns to scale, the distance between IQs are decreasing. It indicates that as firm expands the output, it requires small increases in quantity of L & K to produce the equal increment of output OA > AB > BC > CD. It is cause due to large scale economies (advantages).

2. Constant Returns to scale

Constant Returns to Scale means same proportion of change in output as change in the input. For eg. if all inputs are increased by 25% & output also increases by 25% then constant increase in return to scale will exist. In this case, large scale economies (advantage) & large scale diseconomies (disadvantage) counter balance with each other. It is the linear homogenous prod. function.
In the fig. OR is a scale line. The Isoquants, Q_1 = 10, Q_2 = 20 & Q_3 = 30 indicate three different levels of output. The successive isoquants are equi-distance from each other i.e. OA = AB = BC. This means if both labour & capital are increased in a given proportion then the output expands by the same proportion.

3. Decreasing Returns to Scale

When output increases less than proportionately to increase in inputs it is called decreasing returns to scale. For e.g. if all inputs are increased by 25% & output increased by 20% then decreasing returns to scale will prevail.

In the fig; OR is a scale line. The Isoquants, Q_1 = 10, Q_2 = 20 & Q_3 = 30 indicate three different levels of output. The successive isoquants are larger & larger distance apart on the scale line i.e. OA < AB < BC. This means that more & more of inputs are required to obtain equal increase in output.
1. Cause of Increasing Returns to Scale.
   a. Indivisibility of factors of production.
      Some of the factors such as entrepreneur & heavy machines are indivisible even in the long run. By increasing the scale of output these factors are fully used, their efficiency increases & increasing returns to scale can be obtained. If all the factors are perfectly divisible increasing returns to scale do not operate.
   b. Specialization: In the view of Chamberlin, even if all the factors are perfectly divisible increasing returns to scale will operate because of the greater possibility of specialization of factors such as L & K.
   c. Managerial economics: When single scale of prodt is increased management work can be subdivided in different units or functional basis - such as prodt, finance, marketing, advertisement, etc. They can be entrusted to different specialized personnel & efficiency increases, increasing returns to scale applied.
   d. Dimensional relation: According to Baumol, increasing returns to scale operate due to dimensional rule where the L & K are double the output will become more than double. Output increases more than proportionally when the diameter of water pipe is double the flow of water is more than double.

2. Cause of Constant Returns to Scale
   According to the economist John Robinson, Kaldor, Hicks & Knight, if all the factors are perfectly divisible constant returns to scale operate. If some factors are scarce of
indivisible, constant returns to scale do not even but in the opinion of Chamberlin even if all the factors are divisible due to greater specialization of L & K, as scale of production increased in a fixed proportion increasing returns to scale operate. But empirical evidence shows that in the expansion of a firm, after a certain phase of increasing R.T.S there is a long phase of constant return to scale covering a wide range of output.

3. Causes of Decreasing Returns to Scale (Diseconomies)
   a. Indivisibility of factors: According to Kaldor, decreasing returns to scale occur when indivisible factors such as entrepreneur becomes inefficient and less productive due to any expansion of the scale of production.
   b. Difficulty in management According to Chamberlin, because of the difficulty in supervision & coordination in the production decreasing R.T.S operate.
   c. Scarcity of Natural resources
      When a firm doubles its scale of production, natural resources such as coal deposits, petroleum products etc can’t be doubled.
Chapter 6: Cost & Revenue Curves

1. Total Revenue (TR) \( \rightarrow TR \) is the total amount of money received by the firm from the sales of its own product at a given period of time. \( TR = P \times Q \) \( P = \) Price \( Q = \) Quantity Sold.

2. Average Revenue (AR) \( \rightarrow \) Average revenue is the price per unit. It is obtained by dividing total revenue by the no. of units sold.

\[ AR = \frac{TR}{Q} = \frac{P \times Q}{Q} = P \]

3. Marginal Revenue (MR) \( \rightarrow MR \) is the addition to total revenue from the sale of an additional unit of the commodity.

\[ MR = \frac{\Delta TR}{\Delta Q} \] or, \( MR = TR_{n+1} - TR_n \)

Derivation of Revenue Curve under Perfect Competition.

Perfect competition is the market structure where there are a large number of buyers and sellers producing a homogeneous product. In perfect competition, a firm is a 'price-taker.' A firm can sell whatever quantity it produces at the given price. Therefore, price remains constant at any level of output. The price is determined by market mechanism.

<table>
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<th>Units Sold</th>
<th>AR/Price (RS) = TR/Q</th>
<th>TR = P x Q</th>
<th>MR = TR_{n+1} - TR_n</th>
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